Decoherence time

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I critically assess some published values for decoherence times. I show there are two characteristic times inversely proportional to each other: decoherence time, and probability decay time, with second often mistaken for the first. I present formulas for decoherence and decay times

While decoherence has been extensively studied, surprisingly little is available in terms of numeric results, either calculated from formulas [1, 2] or measured in experiments [3, 4]. The reason for researchers shying away from publishing numbers becomes apparent once these numbers are estimated from widely quoted expressions [5, 1]. The estimated values of decoherence times span an incredible range from $10^{24}(s)$ for a large molecule immersed into cosmic microwave background [1], to $10^{-40}(s)$ for a "canonical" classical object [5] with mass $1g$, characteristic length $1cm$, temperature $300K$, and assumed characteristic "relaxation" time of $1s$. An interval of $10^{-40}(s)$ cannot conceivably correspond to any physical process in a macroscopic object, as no parameter can undergo a change in any meaningful way during such time. To note, it takes 10^{17} times longer, i.e. $0.3 \cdot 10^{-23}(s)$ for light to cross proton radius. Due to time-energy uncertainty, the duration of $10^{-40}(s)$ of a change in object's state would result in energy uncertainty of $10^{6}(J)$, enough to evaporate $1g$ object. Such improbable times call into question the premise decoherence estimates are based on.

Part of the problem, at least for some authors, is that … *no clear, unambiguous and universally accepted definition of coherence (which is supposed to get lost in the process) is available* [6]. Which leads to a confusion as far as which process' characteristic time to take as decoherence time. The common view links coherence with presence of interference terms in density matrix, and their reduction – with decoherence [1]. With this understanding, can the characteristic time of reduction of interference be taken as decoherence time? As I explain below, that is not the case.

The reduction of object's density matrix is achieved by tracing out entangled ancilla system, often assumed to be the environment [1]. As has been shown elsewhere [7], the "tracing out" operation is nothing else but a measurement performed *on* ancilla[1]. The *decoherence time* τ is, therefore, the interval between state preparation[2], and subsequent measurement, even if only on ancilla part of the state. The classical information extracted by measurement on ancilla reduces ambiguity in measurement of entangled object's state [7], i.e. reduces interference terms in density matrix. The reduction of density matrix signifies decay of object's probability distribution.

One would intuitively expect the faster ancilla is measured, the faster will interference decay, but, in fact, the relation between decoherence time τ defined above, and the decay of probability distribution is inverse, i.e. the smaller is τ, the slower is the decay[3]. One can understand this by considering measurements on ancilla as random walk from the surface of generalized *Bloch ball* [8] towards its interior. A point on the surface of Bloch ball corresponds to the initial pure state, and the point in the center corresponds to the complete mixture. The larger are the steps (i.e. the larger is τ), the fewer steps are needed to reduce density matrix, the faster is the probability decay. It can also be schematically proven as follows. The distance from the surface of Bloch ball signifies

[1] and not *by* ancilla, as often mistakenly claimed [1]

[2] the preparation is also measurement, just with a different device, in a different measurement basis

[3] this is also known as quantum Zeno effect [12]

the probability decay. The random walk is described by binomial distribution with probability $p = 1/2$ to make a step in either direction. The variance of binomial distribution after N random steps is given by:

$$\sigma_N^2 = N \cdot p \cdot (1 - p) = N/4 \tag{1}$$

The standard deviation σ_N, multiplied by the length τ of each step, gives the distance gained from the surface of Bloch ball, i.e. the probability decay. If elapsed time is t, then $N = t/\tau$. Using (1):

$$decay = const \cdot \sigma_N \cdot \tau = const \cdot \sqrt{\tau \cdot t} \tag{2}$$

From (2), $t = const \cdot decay^2/\tau$, i.e. the probability decay time t is inversely proportional to decoherence time τ. The expression for probability decay has been obtained in Section 4 of [9]:

$$P(t) = \frac{1}{\Omega} + \left(1 - \frac{1}{\Omega}\right) \cdot exp\left(-\frac{\mathcal{E}_{\boldsymbol{E}}^2 \tau}{h^2} t\right) = \frac{1}{\Omega} + \left(1 - \frac{1}{\Omega}\right) \cdot exp\left(-\frac{t}{t_d}\right) \tag{3}$$

, where t is the elapsed time; t_d is the characteristic decay time; Ω is explained below; $\mathcal{E}_{\boldsymbol{E}}$ is calculated from object's energy spectrum $\{\mathcal{E}_i\}$ as:

$$\mathcal{E}_E^2 = Tr(\boldsymbol{E} \cdot \boldsymbol{E}^\dagger)/2 = -Tr(\boldsymbol{E}^2)/2 \qquad ; \qquad \boldsymbol{E} = \mathcal{E}_{i,j} = \mathcal{E}_i - \mathcal{E}_j \tag{4}$$

A generic formula for decoherence time has been obtained in [9] using Fermi's golden rule:

$$\tau = h\varrho(\mathcal{E}) \tag{5}$$

, where $\varrho(\mathcal{E})$ is density of states, i.e. the number of states per unit energy interval around energy $\mathcal{E}$ of the object. From (3,5) the characteristic probability decay time t_d is:

$$t_d = \frac{h}{\mathcal{E}_{\boldsymbol{E}}^2 \cdot \varrho(\mathcal{E})} \tag{6}$$

For a two-level system, $\mathcal{E}_{\boldsymbol{E}}$ is simply equal to the difference in energy levels: $\mathcal{E}_{\boldsymbol{E}} = \Delta\mathcal{E}$. Assuming no degeneracy, there is 1 state per $\Delta\mathcal{E}$ energy interval, i.e. $\varrho(\mathcal{E}) = 1/\Delta\mathcal{E}$. Then, from (5,6):

$$\tau = t_d = \frac{h}{\Delta\mathcal{E}} \tag{7}$$

Thus, for non-degenerate two-level system, the probability decay time and decoherence time are both equal to Margolus-Levitin bound [10]. A sensible value for decoherence time of $2ns$ has been obtained in [4] from experimental data on spectral linewidth, effectively using formula (7), even though (7) is only applicable to non-degenerate two-level systems.

The value of $\mathcal{E}_{\boldsymbol{E}}$ can be quite large for macroscopic objects. That explains why probability decay time (6) can be incredibly small. One has to keep in mind, the probability decay (3) does not describe a change in any particular object's state. It only means the reduction in correlation between measurements outcomes obtained by different devices when performing measurements on ensemble of identically prepared objects. Each measurement and ensuing [partial] decoherence has characteristic time (5). One cannot measure probability distribution (3) on timescales shorter than (5). On such timescales (3) is a mere abstraction.

I shall now expound on the value of parameter Ω in (3). A measurement outcome is one of possible events $\{i\}$. The measurement event sample $\{n_i\}$ represents the collected information about object's state [9]. Here n_i is the number of occurrences of event i in the sample. There are Ω distinct ways to collect event sample $\{n_i\}$; where Ω equals statistical weight of the sample:

$$\Omega = \frac{N!}{\prod_i n_i!} = e^H \quad (8)$$

, where $H(N, \{n_i\})$ is Boltzmann's entropy of the sample. Different ways to collect the sample (i.e. different event sequences) relate to different correlations between events. One of the ways to collect event sample would have the same sequence of events as that of the measurement sample collected on initial pure state. In full decoherence, all ways to collect event sample $\{n_i\}$ have equal probability $1/\Omega$. Thus, $1/\Omega$ is the minimum probability the measured object is in initial pure state, with confidence (fidelity) provided in [11].

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